

DSP: Lec 9

Realization structures of Digital Filters

"Digital filter structure"

* structure of IIR Filters

Digital filters \Rightarrow discrete time (LTI) systems.

\rightarrow these filters are described by difference equations

\rightarrow These systems can be implemented in hardware or software.

\rightarrow There exist various methods to implement these filters, these methods are called Realization for filter structures.

IIR filters / FIR filters

IIR systems / FIR systems.

IIR \rightarrow Infinite Impulse Response.

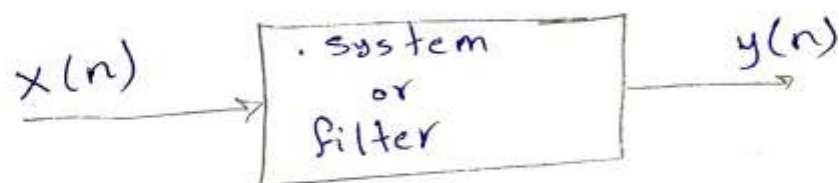
FIR \rightarrow finite

*

* IIR systems / filters

\rightarrow general form for the difference eqn:

$$y(n) + a_1 y(n-1) + a_2 y(n-2) + \dots + a_N y(n-N) \\ = b_0 x(n) + b_1 x(n-1) + \dots + b_M x(n-M)$$



* FIR systems / filters

* general form for the difference equation

$$y(n) = b_0 x(n) + b_1 x(n-1) + \dots + b_M x(n-M)$$

For IIR

~~IIR~~

$$\underline{\underline{Z.T}} \Rightarrow \left[1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N} \right] Y(z)$$

$$= \left[b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M} \right] X(z)$$

$$T.F = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \dots + b_m z^{-m}}{1 + a_1 z^{-1} + \dots + a_N z^{-N}}$$

a - Feedback system.

b - Consists of Poles & Zeros.

c - depend on the Present and Past i/p's and also Past o/p's.

For FIR systems / Filters

Z.T

$$T.F = \frac{Y(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}$$

a - The system isn't feedback system.

b - Consists of zeros only.

c - depend on the Present and Past i/p's.

FIR (IIR في a, b, c) ←

هي الفرق بين الاثنين.

once upon
on an exam

* Recursive Filters \equiv IIR Filters.

* Non Recursive Filters \equiv FIR Filters.

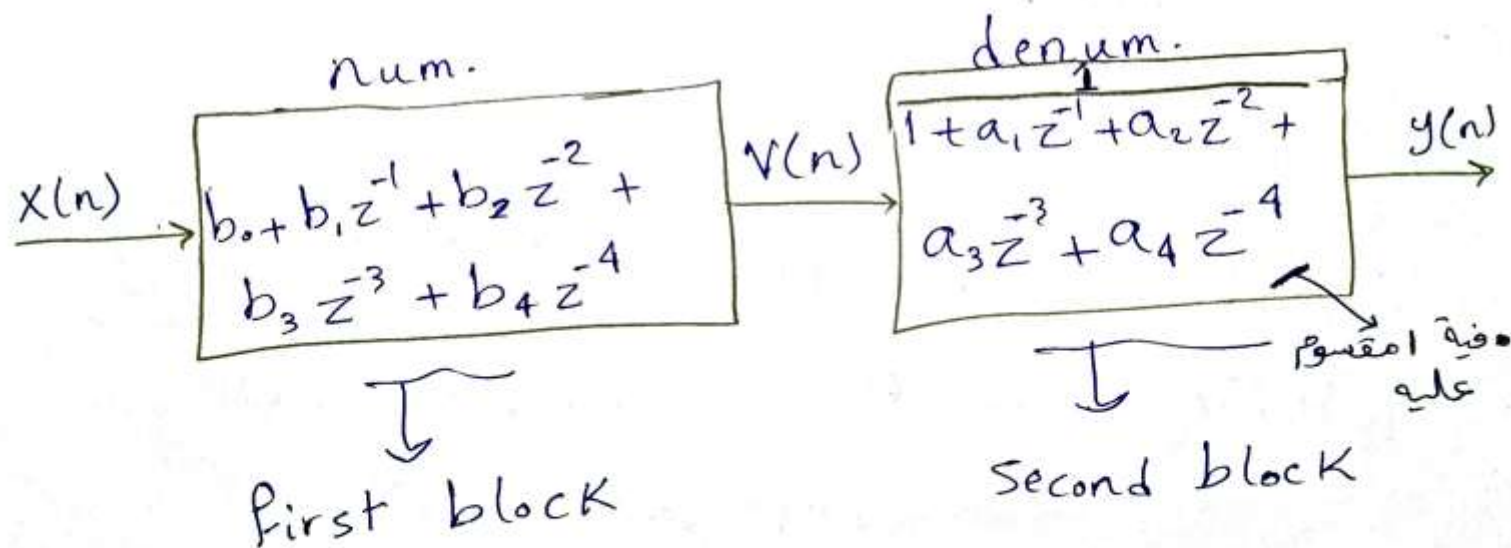
Digital Filter structures (For IIR Filters)

II Direct Form I

$$T.F = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N}}$$

For $N = M = 4$

$$T.F = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}$$

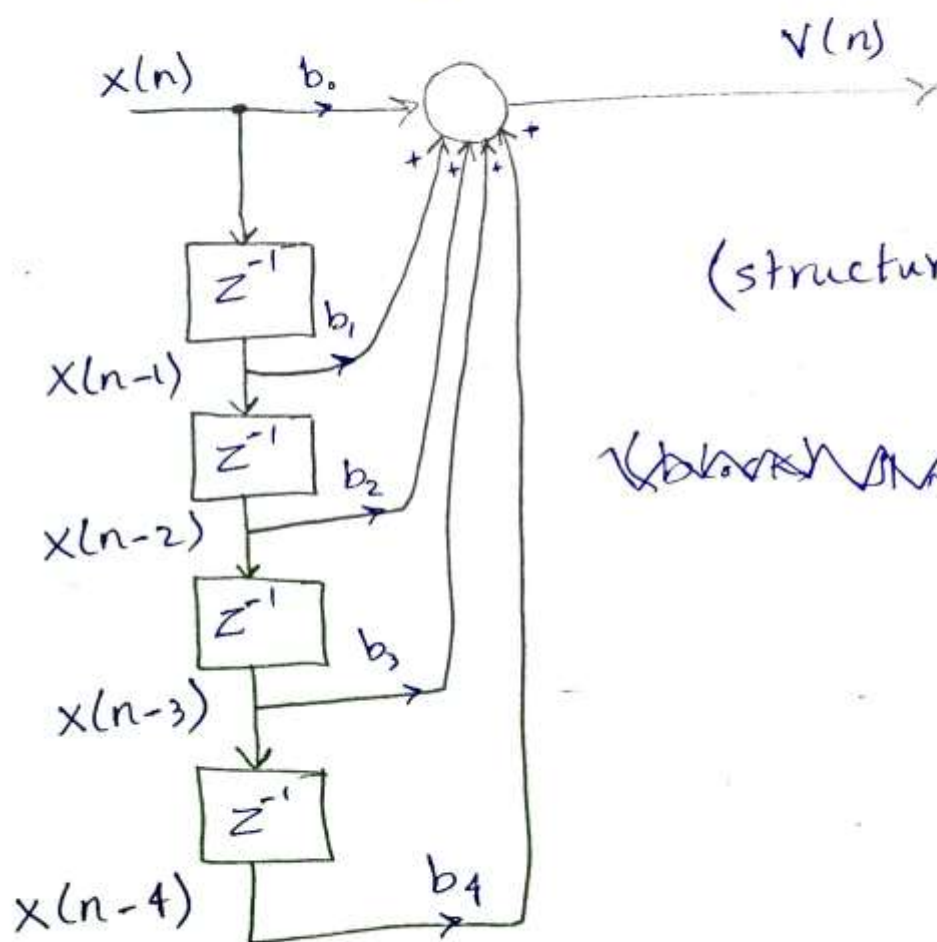


For first block

$$\frac{V(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}$$

$$V(z) = X(z) [b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}] \quad \Downarrow \quad z^{-1} \cdot T$$

$$v(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + b_3 x(n-3) + b_4 x(n-4)$$



هـ جزء من ال (structure)

يعبر عن المسك .

~~من كل جزء جزء~~
منه

* For second block

$$\frac{Y(z)}{V(z)} = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}$$

$$V(z) = [1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}] Y(z)$$

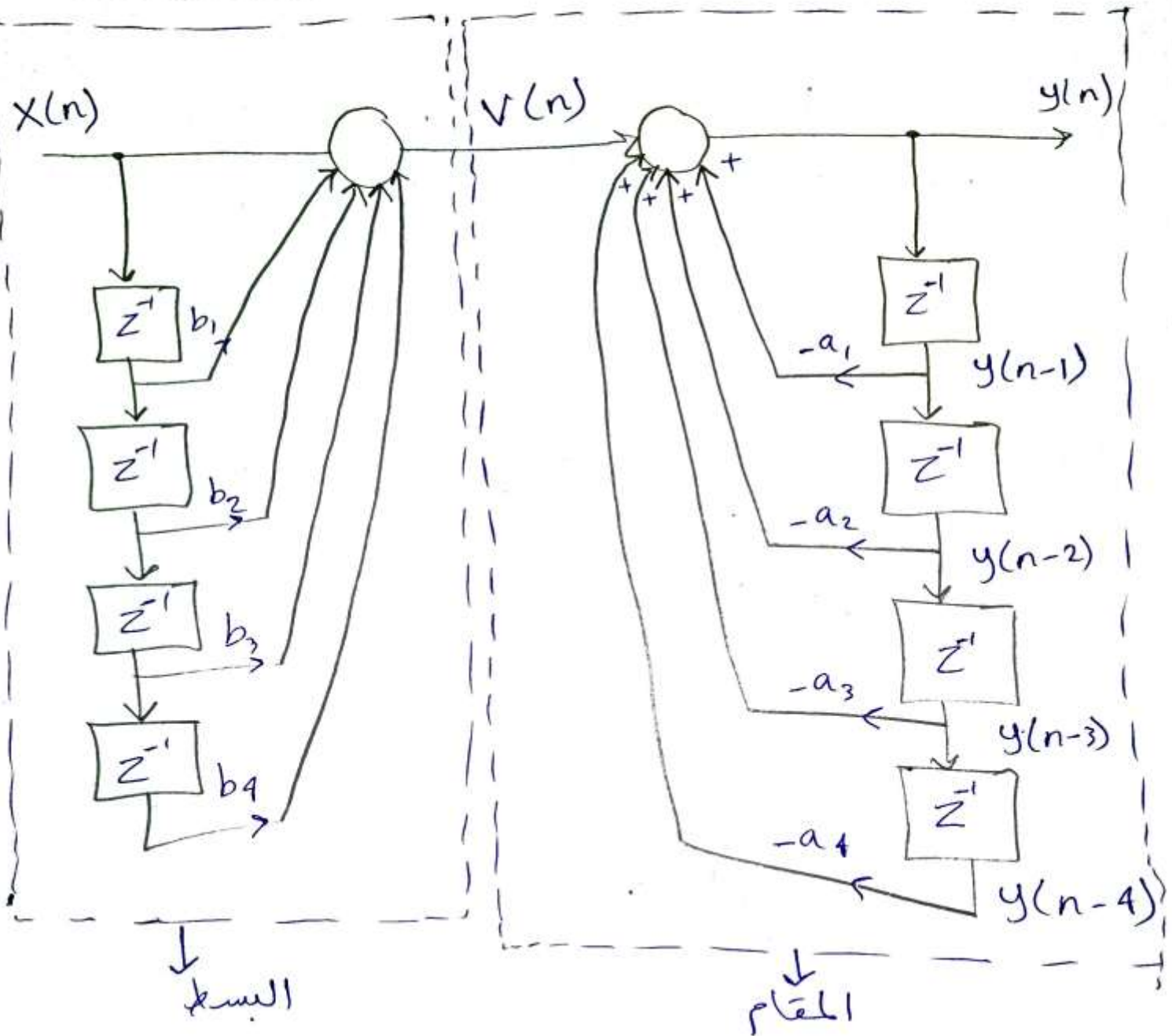
$$Y(z) = V(z) - a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z) - a_3 z^{-3} Y(z) - a_4 z^{-4} Y(z)$$

$$\xrightarrow{z^{-1} \cdot T}$$

$$y(n) = v(n) - a_1 y(n-1) - a_2 y(n-2) - a_3 y(n-3) - a_4 y(n-4)$$

← الرسم كامل البنية، المقام معاً في البنية القادمة.

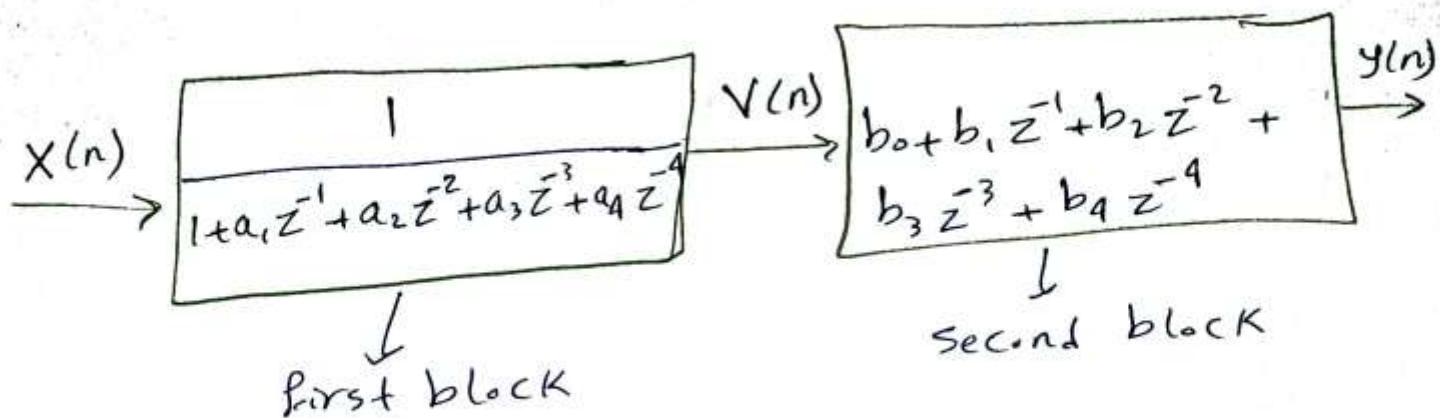
□6□



2 Direct Form II

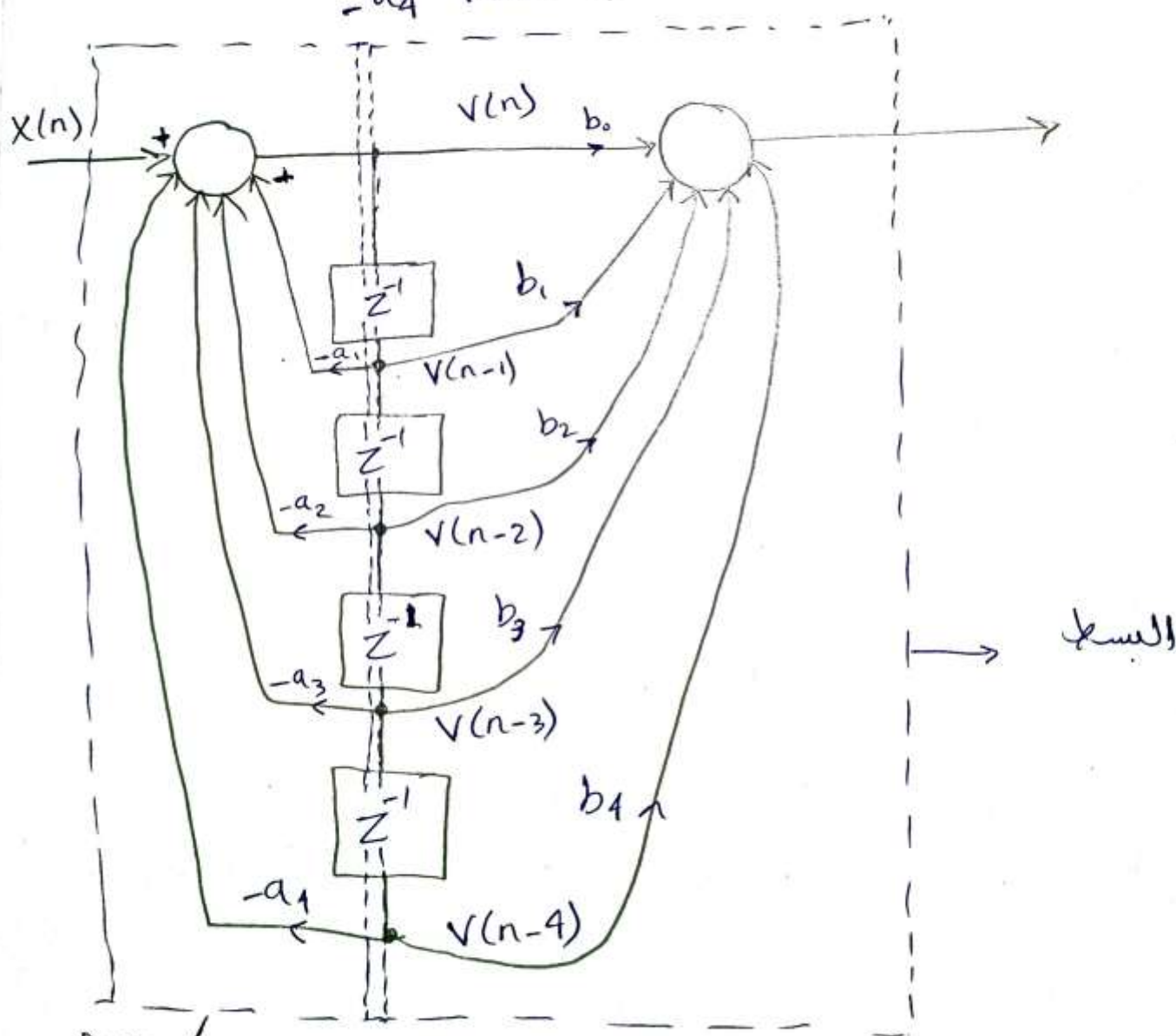
for ex: $N = M = 4$

$$T.F = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}$$



First block

$$v(n) = x(n) - a_1 v(n-1) - a_2 v(n-2) - a_3 v(n-3) - a_4 v(n-4)$$



first block

8

→ second block

$$T.F = \frac{Y(z)}{V(z)} = b_0 + b_1 z^{-1} + \dots + b_4 z^{-4}$$

$$y(n) = b_0 v(n) + b_1 v(n-1) + b_2 v(n-2) + b_3 v(n-3) + b_4 v(n-4)$$

← الرسمة كاملة في الصفحة السابقة .

Ex For the following Digital Filter

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{(1 + z^{-1})(1 + 0.4 z^{-1})(1 + 0.9 z^{-1})}$$

Draw the corresponding Direct form I & II

$$H(z) = \frac{1 + z^{-2}}{1 + 2.3 z^{-1} + 1.66 z^{-2} + 0.36 z^{-3}}$$

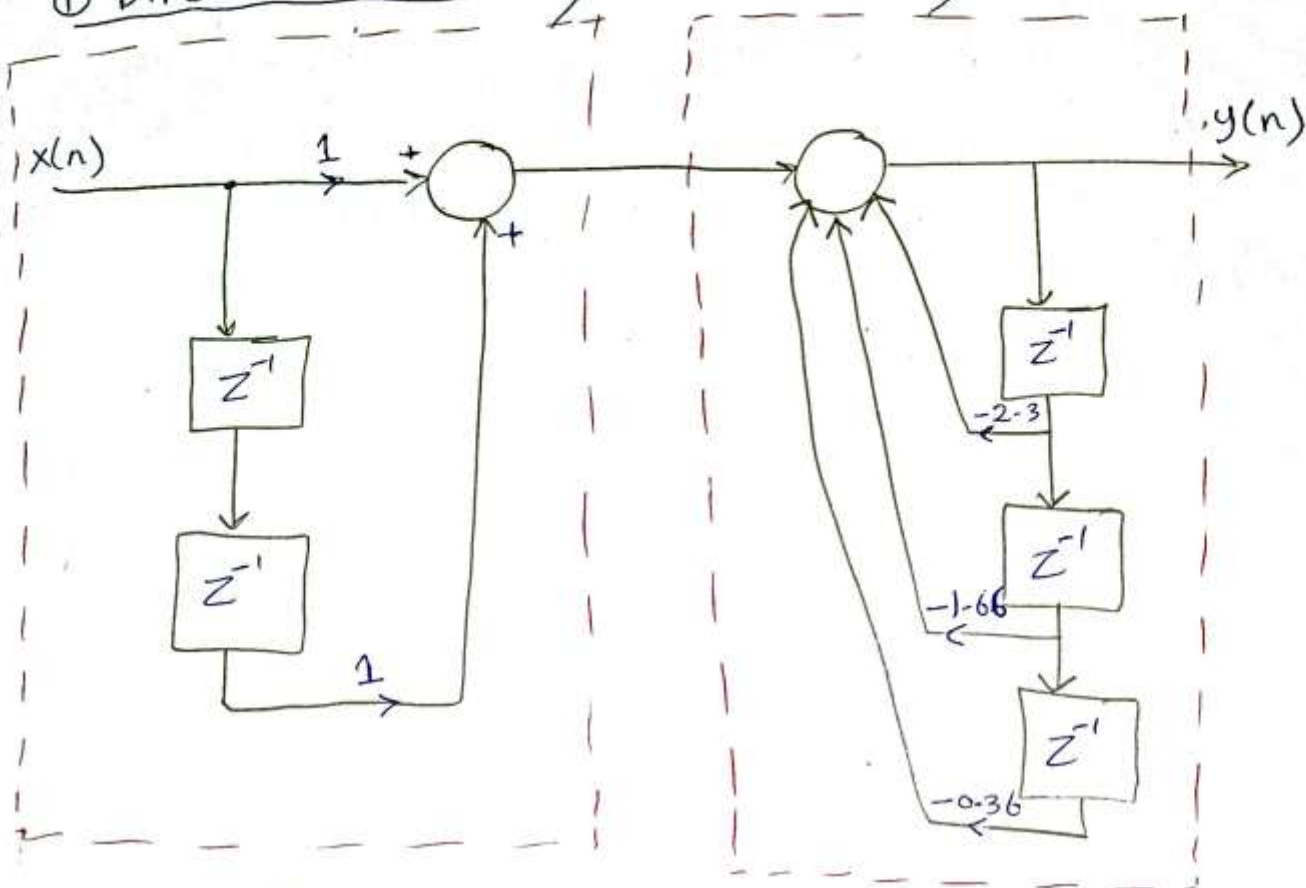
لازم تكون
ب. 1.

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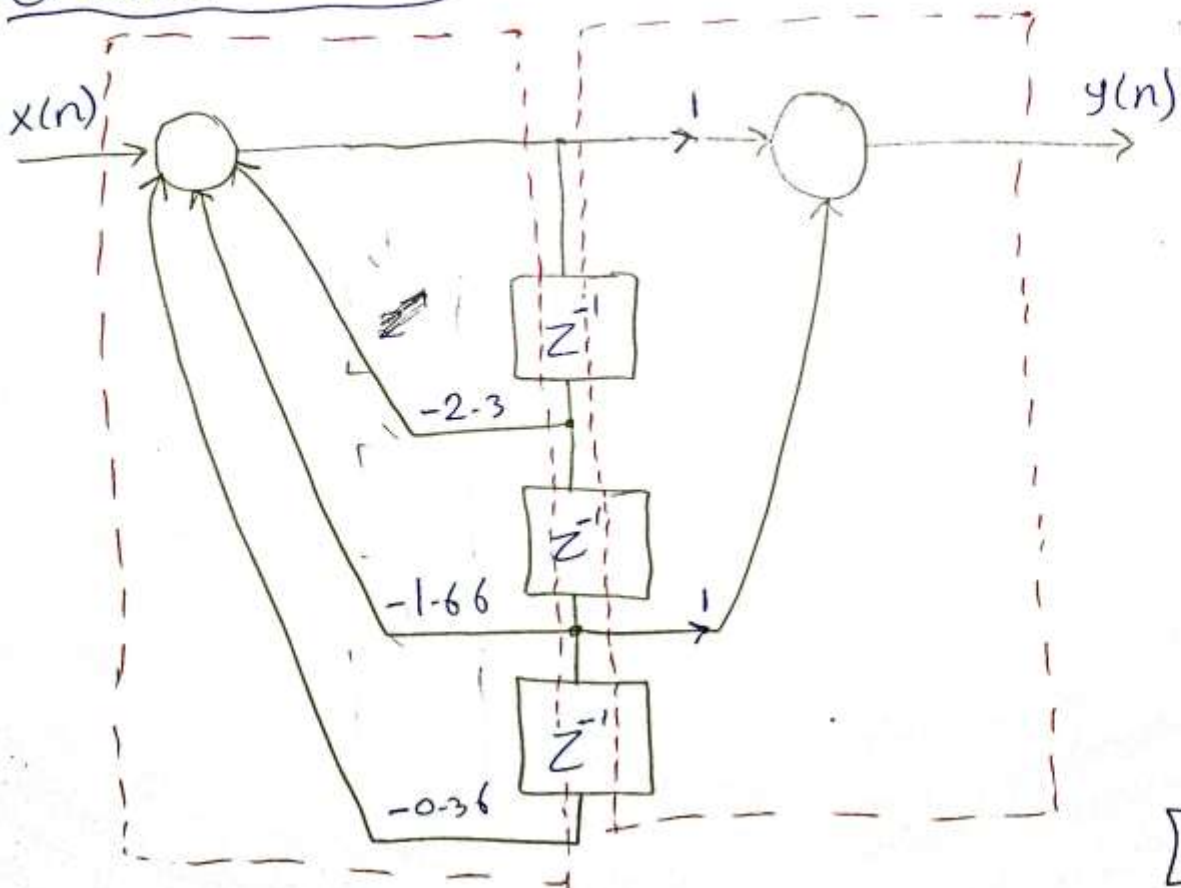
① Direct Form I

البسط

المقام



② Direct Form II



EX 2

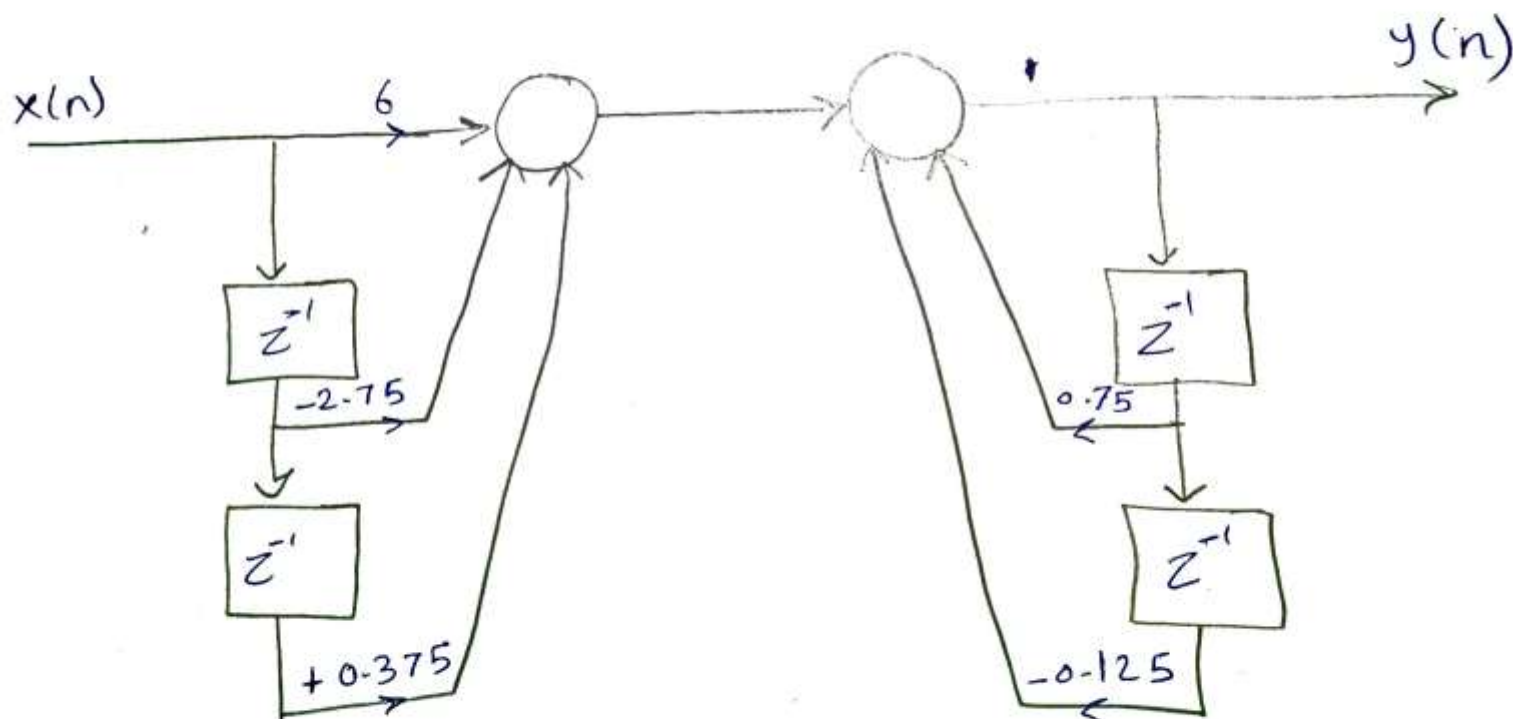
$$H(z) = 3 + \frac{4z}{z-0.5} + \frac{z}{z-0.25}$$

$$= \frac{6z^2 - 2.75z + 0.375}{z^2 - 0.75z + 0.125}$$

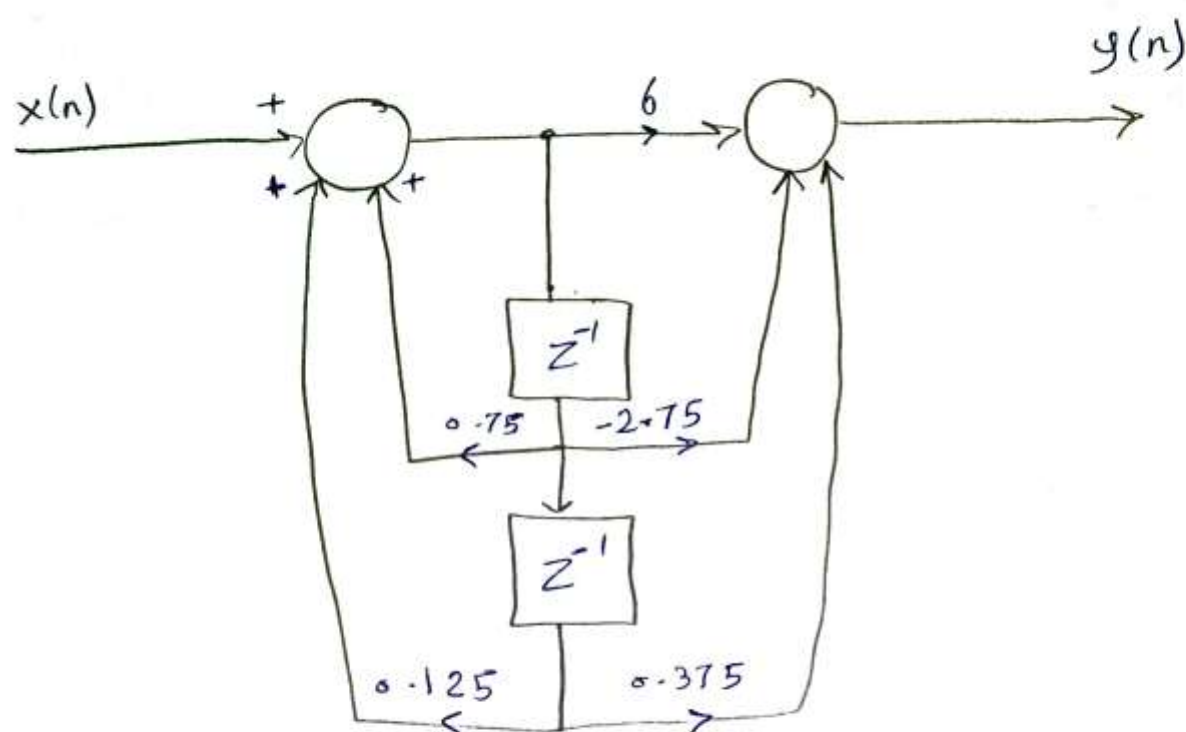
$$\times \frac{z^{-2}}{z^{-2}}$$

$$= \frac{6 - 2.75z^{-1} + 0.375z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

* Direct Form I



* Direct Form II



[3] Parallel Form

$$T.F = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}$$

$$= \frac{\quad}{(\quad) (\quad) (\quad) \cdots (\quad)} \quad \Downarrow \text{P.F.}$$

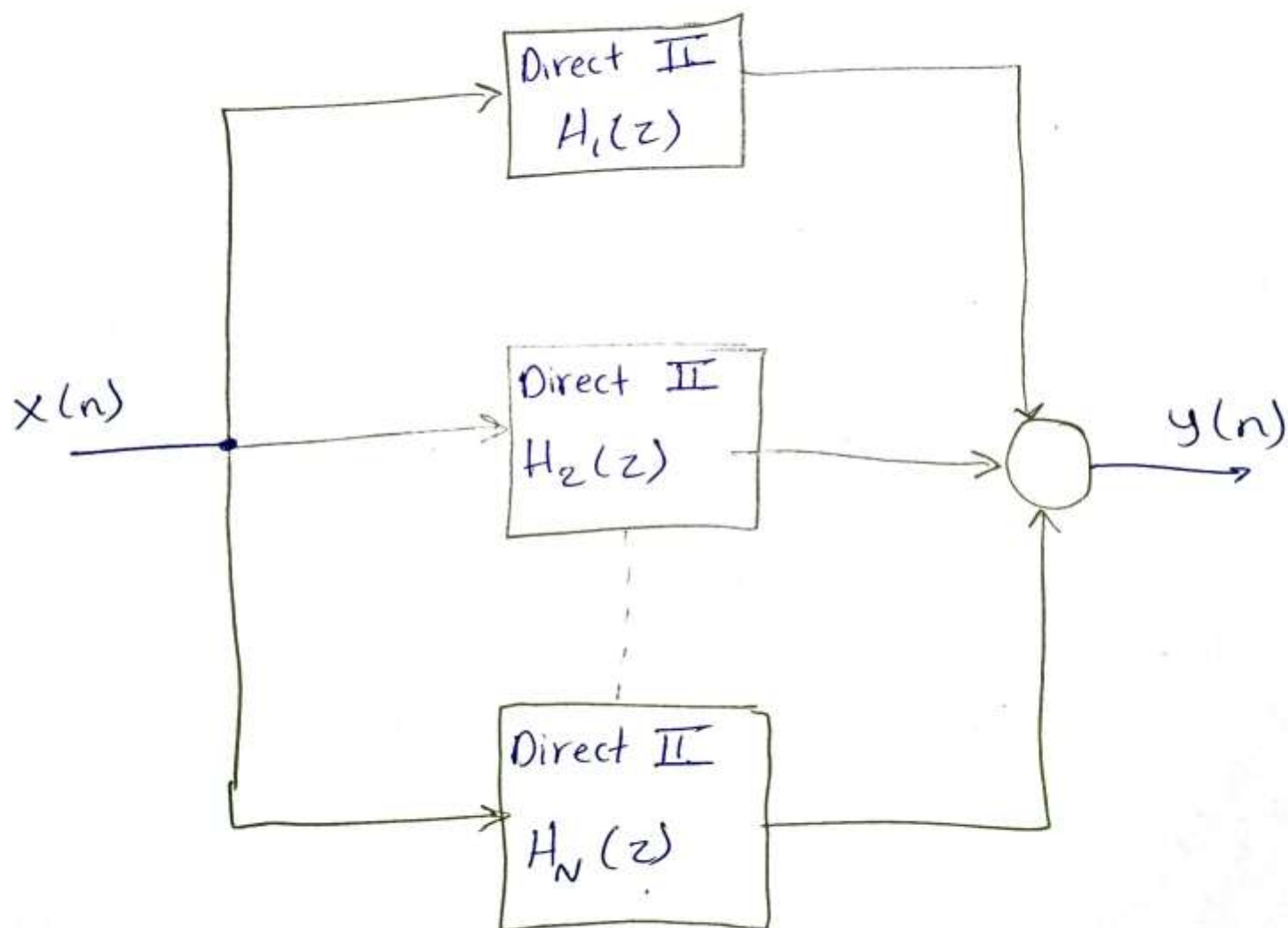
First or ⁴second order

$$= \frac{A_1}{(\quad)} + \frac{A_2}{(\quad)} + \cdots$$

$$T.F = H_1(z) + H_2(z) + \dots + H_N(z)$$

Direct Form I or II

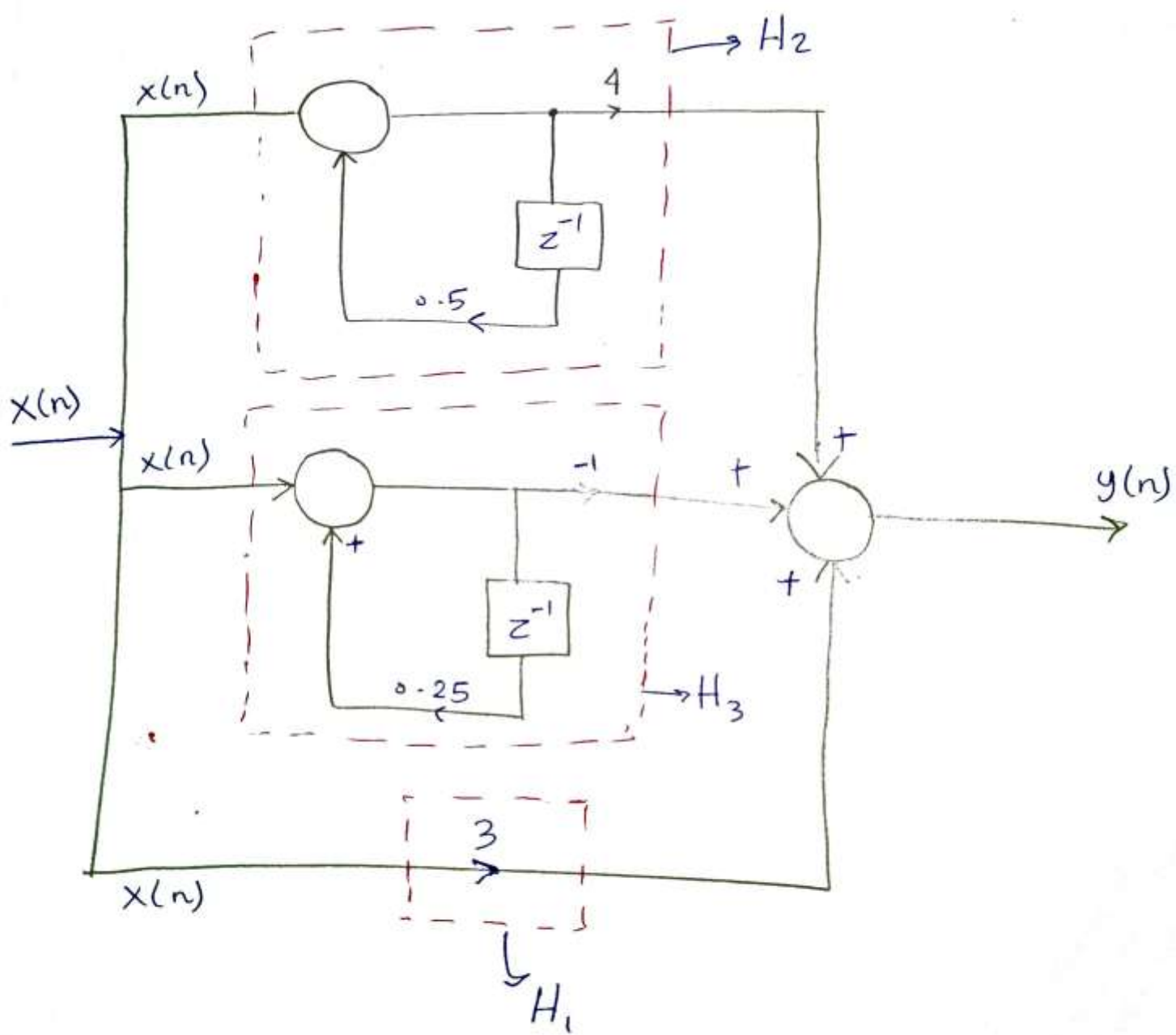
هنا نحلل المقام ثم نقسم للمقام بواسطة (Partial Fraction)
ثم نفس كل جزء ب $H(z)$ مثلاً ونحسب (Direct Form)
(II or I) ونجمع معاً فتدعى الـ (Parallel Form)



Ex $H(z) = 3 + \frac{4z}{z-0.5} - \frac{z}{z-0.25}$

Realize the system using Parallel Form:-

$$H(z) = \underbrace{3}_{H_1} + \underbrace{\frac{4}{1-0.5z^{-1}}}_{H_2} - \underbrace{\frac{1}{1-0.25z^{-1}}}_{H_3}$$



Ex

$$H(z) = \frac{1 + z^{-2}}{(1 + z^{-1})(1 + 0.4z^{-1})(1 + 0.9z^{-1})}$$

Draw the structure form in parallel form.

$$\boxed{\text{Put } x = z^{-1}}$$

$$H(z) = \frac{1 + x^2}{(1+x)(1+0.4x)(1+0.9x)} = \frac{A_1}{1+x} + \frac{A_2}{1+0.4x} + \frac{A_3}{1+0.9x}$$

$$A_1 = 33.33, \quad A_2 = 3.87, \quad A_3 = -36.2$$

$$\text{Put } x \rightarrow z^{-1}$$

■

$$H(z) = \underbrace{\frac{33.33}{1 + z^{-1}}}_{H_1} + \underbrace{\frac{3.87}{1 + 0.4z^{-1}}}_{H_2} - \underbrace{\frac{36.2}{1 + 0.9z^{-1}}}_{H_3}$$

